

**Chapter 9**  
**Areas of Parallelograms and Triangles**

**Exercise No. 9.1**

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**Multiple Choice Questions:**

Write the correct answer in each of the following:

**1. The median of a triangle divides it into two**

- (A) triangles of equal area
- (B) congruent triangles
- (C) right triangles
- (D) isosceles triangles

**Solution:**

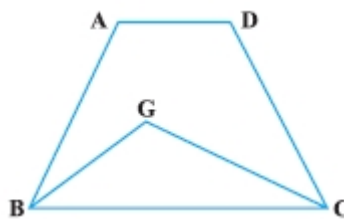
A median of a triangle divides it into two triangles of equal area.  
Hence, the correct option is (A).

**2. In which of the following figures, you find two polygons on the same base and between the same parallels?**

(A)



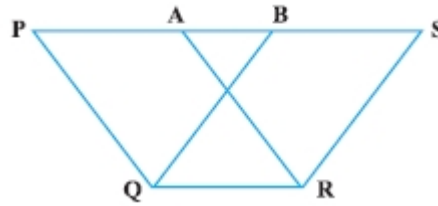
(B)



(C)



(D)



**Solution:**

In figure (d), we find two polygons (PQRA and BQRS) on the same base and between the same parallels.

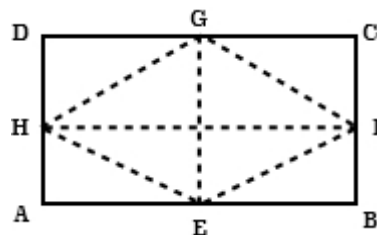
Hence, the correct option is (D).

**3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is:**

- (A) a rectangle of area  $24 \text{ cm}^2$ .
- (B) a square of area  $25 \text{ cm}^2$ .
- (C) a trapezium of area  $24 \text{ cm}^2$ .
- (D) a rhombus of area  $24 \text{ cm}^2$ .

**Solution:**

According to the question,

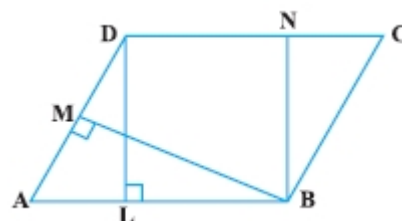


ABCD is a rectangle and E, F, G and H are the mid-point of the sides AB, BC, CD and DA respectively. The figure formed is rhombus whose area:

$$\begin{aligned}
 &= \frac{1}{2} \times EG \times FH \\
 &= \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm} \\
 &= 24 \text{ cm}^2
 \end{aligned}$$

Hence, the correct option is (D).

**4. In Fig., the area of parallelogram ABCD is:**



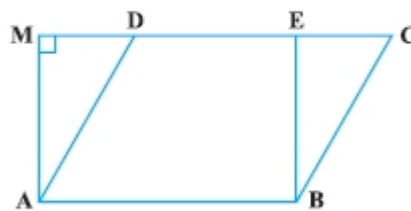
- (A)  $AB \times BM$

- (B)  $BC \times BN$   
 (C)  $DC \times DL$   
 (D)  $AD \times DL$

**Solution:**

Area of parallelogram = Base  $\times$  Corresponding altitude  
 $= AB \times DL = DC \times DL$  [Since,  $AB = DC$  (opposite side of a parallelogram)]  
 Hence, the correct option is (C).

**5. In Fig., if parallelogram ABCD and rectangle ABEF are of equal area, then:**



- (A) Perimeter of ABCD = Perimeter of ABEM  
 (B) Perimeter of ABCD < Perimeter of ABEM  
 (C) Perimeter of ABCD > Perimeter of ABEM  
 (D) Perimeter of ABCD =  $\frac{1}{2}$  (Perimeter of ABEM)

**Solution:**

If parallelogram ABCD and rectangle ABEF are of equal area then perimeter of ABCD > Perimeter of ABEM because:

As we know that, the perpendicular distance between two parallel sides of a parallelogram is always less than the length of the other parallel sides.

$BE < BC$  and  $AM < AD$ .

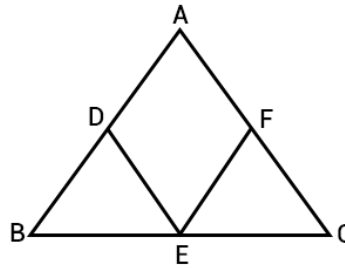
Hence, the correct option is (C).

**6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to**

- (A)  $\frac{1}{2} \text{ar}(\text{ABC})$   
 (B)  $\frac{1}{3} \text{ar}(\text{ABC})$   
 (C)  $\frac{1}{4} \text{ar}(\text{ABC})$   
 (D)  $\text{ar}(\text{ABC})$

**Solution:**

We know that, median of a triangle divides it into two triangle of equal area.



$$\text{So, area}(\triangle ADE) = \text{area}(\triangle BDE) \quad \dots(\text{I})$$

$$\text{area}(\triangle AEF) = \text{area}(\triangle EFC) \quad \dots(\text{II})$$

Now, AE is the diagonal of a parallelogram ADEF. That is divides it into two triangles of equal area.

$$\text{So, area}(\triangle ADE) = \text{area}(\triangle AFE) \quad \dots(\text{III})$$

Now, from equation (I), (II), and (III), get:

$$\text{area}(\triangle ADE) = \text{area}(\triangle BDE) = \text{area}(\triangle AFE) = \text{area}(\triangle EFC)$$

$$\text{Hence, area}(\triangle ADEF) = \frac{1}{2} \text{area}(\triangle ABC)$$

Therefore, the correct option is (A).

**7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is**

- (A) 1 : 2
- (B) 1 : 1
- (C) 2 : 1
- (D) 3 : 1

**Solution:**

As we know that parallelogram on the same or equal bases and between the same parallels are equal in area.

So, the ratio of these area is 1:1.

Hence, the correct option is (B).

**8. ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD**

- (A) is a rectangle
- (B) is always a rhombus
- (C) is a parallelogram
- (D) need not be any of (A), (B) or (C)

**Solution:**

The quadrilateral ABCD need not be any of rectangle, rhombus and parallelogram because if quadrilateral ABCD is a square then its diagonal AC also divides it into two parts which are equal in area.

Hence, the correct option is (D).

**9. If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of the area of the triangle to the area of parallelogram is**

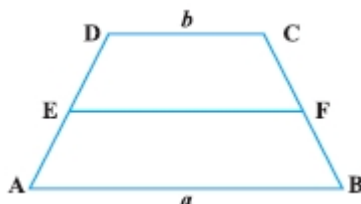
- (A) 1 : 3
- (B) 1 : 2
- (C) 3 : 1
- (D) 1 : 4

**Solution:**

As we know that, if a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram. Therefore, the ratio of the area of the triangle to the area of parallelogram is 1:2.

Hence, the correct option is (B).

**10. ABCD is a trapezium with parallel sides  $AB = a$  cm and  $DC = b$  cm. E and F are the mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is**



- (A)  $a : b$
- (B)  $(3a + b) : (a + 3b)$
- (C)  $(a + 3b) : (3a + b)$
- (D)  $(2a + b) : (3a + b)$

**Solution:**

Given:

ABCD is a trapezium with parallel sides such that  $AB \parallel DC$  and  $AB = a$  cm and  $DC = b$  cm. E and F are the mid-points of the non-parallel sides that are AD and BC. So,

$$EF = \frac{1}{2}(a + b)$$

ABEF and EFCD are also trapezium.

$$\text{area}(ABEF) = \frac{1}{2} \left[ \frac{1}{2}(a + b) + a \right] \times h = \frac{h}{4}(3a + b)$$

$$\text{area}(EFCD) = \frac{1}{2} \left[ b + \frac{1}{2}(a + b) \right] \times h = \frac{h}{4}(a + 3b)$$

So,

$$\frac{\text{area}(ABEF)}{\text{area}(EFCD)} = \frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)} = \frac{3a+b}{a+3b}$$

So, the required ratio is  $(3a+b):(a+3b)$ .  
Hence, the correct option is (B).



## Exercise No. 9.2

### Short Answer Questions with Reasoning:

Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If  $\text{ar}(\triangle XCD) = 24 \text{ cm}^2$ , then  $\text{ar}(\triangle ABC) = 24 \text{ cm}^2$ .

**Solution:**

Given in the question, ABCD is a parallelogram and X is the mid-point of AB.

So,  $\text{area}(\triangle ABCD) = \text{area}(\triangle XCD) + \text{area}(\triangle XBC) \dots \text{(I)}$

Now, diagonal AC of a parallelogram divides it into two triangles of equal area.

$\text{area}(\triangle ABCD) = 2\text{area}(\triangle ABC) \dots \text{(II)}$

Similarly, X is the mid-point of AB, So,

$\text{area}(\triangle CXB) = \frac{1}{2} \text{area}(\triangle ABC) \dots \text{(III)}$  [Median divides the triangle in two triangles of equal area]

$2\text{area}(\triangle ABC) = 24 + \frac{1}{2} \text{area}(\triangle ABC)$  [By using equation (I), (II) and (III)]

Now,  $2\text{area}(\triangle ABC) - \frac{1}{2} \text{area}(\triangle ABC) = 24$

$\frac{3}{2} \text{area}(\triangle ABC) = 24$

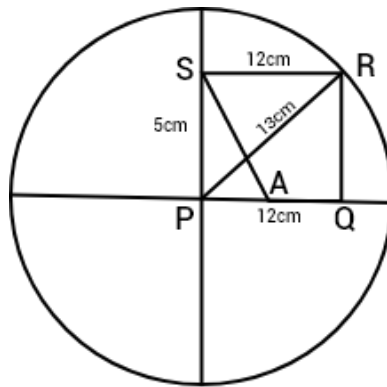
Therefore,  $\text{area}(\triangle ABC) = \frac{2 \times 24}{3} = 16 \text{ cm}^2$ .

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then  $\text{ar}(\triangle PAS) = 30 \text{ cm}^2$

**Solution:**

Given: A is any point on PQ. Since,  $PA < PQ$





Now, area of triangle PQR is:

$$\text{area}(\triangle PQR) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{So, area}(\triangle PQR) = \frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 12 \times 5 = 30\text{cm}^2 \text{ [PQRS is a rectangle, } RQ=SP=5 \text{ cm]}$$

As  $PA < PQ$

So,  $\text{area}(\triangle PAS) < \text{area}(\triangle PQR)$

$$\text{Or area}(\triangle PAS) < 30\text{cm}^2 \quad [\text{area}(\triangle PQR) = 30\text{cm}^2]$$

Hence, the given statement is false.

**3. PQRS is a parallelogram whose area is  $180 \text{ cm}^2$  and A is any point on the diagonal QS. The area of  $\triangle ASR = 90 \text{ cm}^2$ .**

**Solution:**

Given: PQRS is a parallelogram.

As we know that diagonal of a parallelogram divides parallelogram into two triangles of equal area.

So,

$$\begin{aligned} \text{area}(\triangle QRS) &= \frac{1}{2} \text{area}(PQRS) \\ &= \frac{1}{2} \times 180 = 90\text{cm}^2 \end{aligned}$$

Now, A is any point on SQ. So,

$$\text{area}(\triangle ASR) < \text{area}(\triangle QRS)$$

$$\text{Therefore, area}(\triangle ASR) < 90\text{cm}^2$$

Hence, the given statement is false.

**4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then  $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$ .**

**Solution:**



Given:  $\triangle ABC$  and  $\triangle BDE$  are two equilateral triangles.

Suppose that each sides of triangle ABC be  $x$ .

Similarly, D is the mid-point of BC. So, each side of triangle BDE is  $\frac{x}{2}$ .

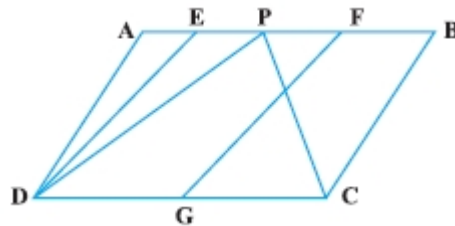
Now,

$$\frac{\text{area}(\triangle BDE)}{\text{area}(\triangle ABC)} = \frac{\frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4} \times x^2} = \frac{x^2}{4x^2} = \frac{1}{4}$$

Therefore,  $\text{area}(\triangle BDE) = \frac{1}{4} \text{area}(\triangle ABC)$ .

Hence, the given statement is true.

**5. In Fig., ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then  $\text{ar}(\triangle DPC) = \frac{1}{4} \text{ar}(\triangle EFGD)$ .**



### Solution:

As triangle DPC and parallelogram ABCD are on same base DC and between the same parallels AB and DC. So,

$$\text{area}(\triangle DPC) = \frac{1}{2} \text{area}(ABCD) \quad \dots(I)$$

Now,

$$\frac{\text{area}(EFGD)}{\text{area}(ABCD)} = \frac{DG \times h}{DC \times h} = \frac{DG}{2DG} = \frac{1}{2} \quad (\text{G is the mid-point of DC})$$

$$\text{Implies that, } \text{area}(EFGD) = \frac{1}{2} \text{area}(ABCD)$$

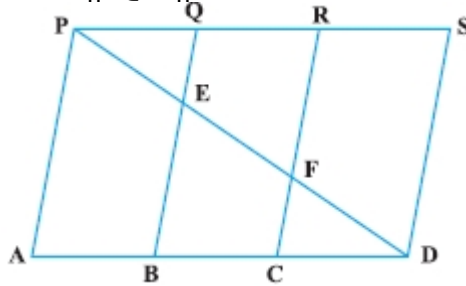
$$\text{So, } \text{area}(DPC) = \text{area}(EFGD) \quad [\text{From equation (I)}]$$

Hence, the given statement is false.

## Exercise No. 9.3

### Short Answer Questions:

1. In Fig., PSDA is a parallelogram. Points Q and R are taken on PS such that  $PQ = QR = RS$  and  $PA \parallel QB \parallel RC$ . Prove that  $\text{ar}(\triangle PQE) = \text{ar}(\triangle CFD)$ .



#### Solution:

Given: PSDA is a parallelogram. Points Q and R are taken on PS such that  $PQ = QR = RS$  and  $PA \parallel QB \parallel RC$ .

To prove that  $\text{ar}(\triangle PQE) = \text{ar}(\triangle CFD)$ .

Proof:  $PS = AD$  [opposite side of a parallelogram]

$$\frac{1}{3} PS = \frac{1}{3} AD$$

$$PQ = CD \quad \dots(I)$$

Similarly,  $PS \parallel AD$  and  $QB$  cut them. So,

$$\angle PQE = \angle CBE \quad [\text{Alternate angles}] \dots(II)$$

Again,  $QB \parallel RC$  and  $AD$  cut them,

$$\angle QBD = \angle RCD \quad [\text{Corresponding angle}] \quad \dots(III)$$

So,  $\angle PQE = \angle FCD$  ... (IV) [From (II) and (III),  $\angle CBE$  and  $\angle QBD$  are same and  $\angle RCD$  and  $\angle FCD$  are same]

Now, in triangle  $PQE$  and triangle  $CFD$ ,

$$\angle PQE = \angle CDF \quad [\text{Alternate angle}]$$

$$PQ = CD \quad [\text{From equation (I)}]$$

$$\angle QPE = \angle FCD \quad [\text{From equation (IV)}]$$

$$\triangle PQE \cong \triangle CFD \quad [\text{By ASA congruence rule}]$$

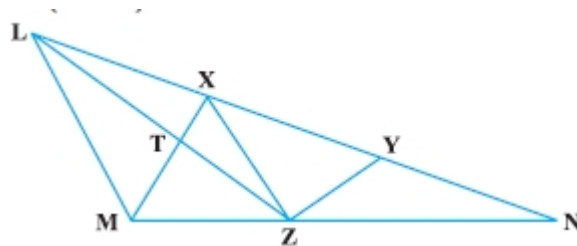
Hence,  $\text{ar}(\triangle PQE) = \text{ar}(\triangle CFD)$ . [Congruent triangles are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that  $LX = XY = YN$ . Through X, a line is drawn parallel to LM to meet MN at Z (See Fig.).

Prove that

$$\text{ar}(\triangle LZY) = \text{ar}(\triangle MZYX)$$





### Solution:

Prove that  $ar(LZY) = ar(MZYX)$

Proof: As  $\triangle LXZ$  and  $\triangle XMZ$  are on the same base and between the same parallels LM and XZ.

$$ar(\triangle LXZ) = ar(\triangle XMZ)$$

Now, adding  $ar(\triangle XYZ)$  to both sides of (I), get:

$$ar(\triangle LXZ) + ar(\triangle XYZ) = ar(\triangle XMZ) + ar(\triangle XYZ)$$

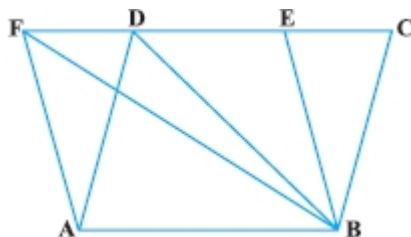
$$ar(\triangle LZY) = ar(MZYX)$$

**3. The area of the parallelogram ABCD is  $90 \text{ cm}^2$  (see Fig.). Find**

**(i)  $ar(ABEF)$**

**(ii)  $ar(\triangle ABD)$**

**(iii)  $ar(\triangle BEF)$**



### Solution:

(i) As we know that parallelogram on the same base and between the same parallels are equal in area.

$$ar(ABEF) = ar(ABCD)$$

$$\text{Hence, } ar(ABEF) = ar(ABCD) = 90\text{cm}^2.$$

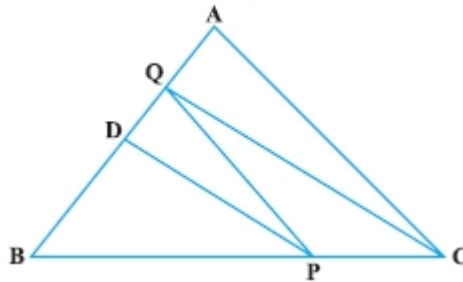
(ii)  $ar(\triangle ABD) = \frac{1}{2} ar(ABCD)$  [A diagonal of a parallelogram divides the parallelogram in two triangle of equal area]

$$= \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$$

(iii)  $ar(\triangle BEF) = \frac{1}{2} ar(ABEF) = \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$

**4. In  $\triangle ABC$ , D is the mid-point of AB and P is any point on BC. If  $CQ \parallel PD$  meets AB in Q (Fig.), then prove that**

$$\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$$

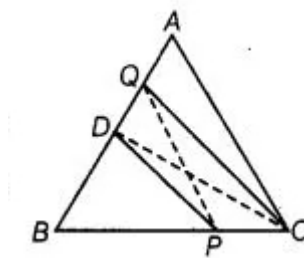


**Solution:**

Given in triangle ABC, D is the mid-point of AB and P is any point on BC.  
 $CQ \parallel PD$  means AB in Q.

To prove that  $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$

Construction: Join PQ and CD.



Proof:

As we know that median of a triangle divides it into two triangles of equal area. So,

$$\text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad \dots(I)$$

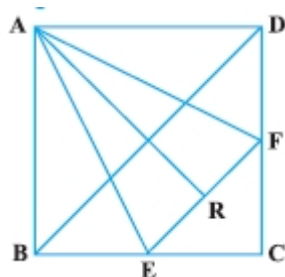
Also, we know that triangles on the same base and between the same parallels are equal in area.  
 So,

$\text{ar}(\triangle DPQ) = \text{ar}(\triangle DPC)$  [Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

$$\text{ar}(\triangle DPQ) + \text{ar}(\triangle DPB) = \text{ar}(\triangle DPC) + \text{ar}(\triangle DPB)$$

$$\text{Hence, } \text{ar}(\triangle BPQ) = \text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC).$$

**5. ABCD is a square. E and F are respectively the midpoints of BC and CD. If R is the mid-point of EF (Fig.), prove that  $\text{ar}(\triangle AER) = \text{ar}(\triangle AFR)$**



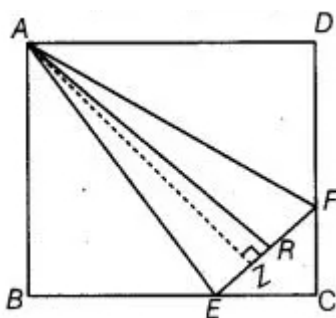
**Solution:**

Given: ABCD is a square. E and F are respectively the midpoints of BC and CD. Also, R is the mid-point of EF.

To prove that  $ar(\triangle AER) = ar(\triangle AFR)$

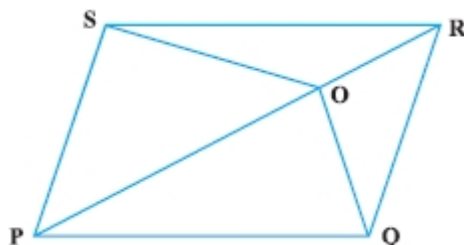
Construction: Draw  $AN \perp EF$

Proof:



$$\begin{aligned}
 ar(\triangle AER) &= \frac{1}{2} \times \text{Base} \times \text{Height} \\
 &= \frac{1}{2} \times ER \times AN \\
 &= \frac{1}{2} \times FR \times AN \quad [\text{R is the mid-point of EF so } ER = FR] \\
 &= ar(\triangle AFR)
 \end{aligned}$$

**6. O is any point on the diagonal PR of a parallelogram PQRS (Fig.). Prove that  $ar(\triangle PSO) = ar(\triangle PQO)$ .**

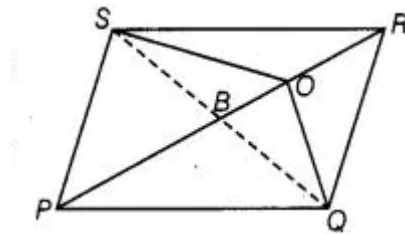


**Solution:**

Given: O is any point on the diagonal PR of a parallelogram PQRS.

To prove that  $ar(\triangle PSO) = ar(\triangle PQO)$ .

Construction: Join SQ which intersects PR at B.



Proof: B is the mid-point of SQ because diagonal of a parallelogram bisect each other. See the above figure, PB is a median of  $\triangle QPS$  and as we know that a median of a triangle divides it into two triangles of equal area.

$$ar(\triangle BPQ) = ar(\triangle BPS) \quad \dots(I)$$

Similarly, OB is the median of  $\triangle OSQ$ .

$$ar(\triangle OBQ) = ar(\triangle OBS) \quad \dots(II)$$

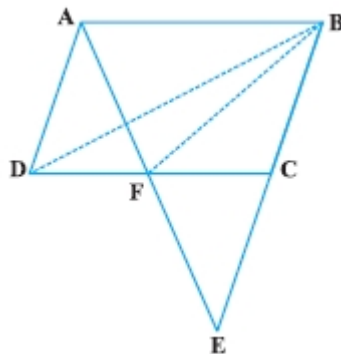
Now, adding equation (I) and (II), get:

$$ar(\triangle BPQ) + ar(\triangle OBQ) = ar(\triangle BPS) + ar(\triangle OBS)$$

$$ar(\triangle PQO) = ar(\triangle PSO)$$

**7. ABCD is a parallelogram in which BC is produced to E such that CE = BC (Fig.). AE intersects CD at F. AE intersects CD at F.**

**If  $ar(\triangle DFB) = 3 \text{ cm}^2$ , find the area of the parallelogram ABCD.**



**Solution:**

Given: ABCD is a parallelogram in which BC is produced to E such that CE = BC. C is the mid-point BE and  $ar(\triangle DFB) = 3 \text{ cm}^2$ .

In triangle ADF and triangle EFC,

$$\angle DAF = \angle CEF \quad [\text{Alternate interior angle}]$$

$$AD = CE \quad [AD = BC = CE]$$

$$\angle ADF = \angle FCE \quad [\text{Alternate interior angle}]$$

So,  $\triangle ADF \cong \triangle ECF$  [By SAS rule of congruence]

Now,  $\triangle ADF \cong \triangle ECF$  [By SAS rule of congruence]



$$DF = CF \quad [\text{CPCT}]$$

As BF is the median of triangle BCD.

$$ar(\triangle BDF) = \frac{1}{2} ar(\triangle BCD) \quad \dots(I) \quad [\text{Median divides a triangle into two triangles of equal area}]$$

As we know that a triangle and parallelogram are on the same base and between the same parallels then the area of the triangle is equal to half the area of the parallelogram.

$$ar(\triangle BCD) = \frac{1}{2} ar(ABCD) \quad \dots(II)$$

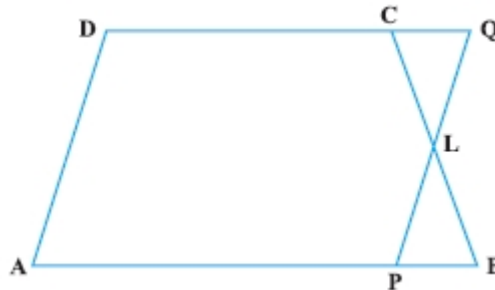
$$ar(\triangle BDF) = \frac{1}{2} \left\{ \frac{1}{2} ar(ABCD) \right\} \quad [\text{By equation (I)}]$$

$$3\text{cm}^2 = \frac{1}{4} ar(ABCD)$$

$$ar(ABCD) = 12\text{cm}^2$$

Hence, the area of the parallelogram is  $12\text{cm}^2$ .

**8. In trapezium ABCD,  $AB \parallel DC$  and L is the mid-point of BC. Through L, a line PQ  $\parallel AD$  has been drawn which meets AB in P and DC produced in Q (Fig.). Prove that  $ar(ABCD) = ar(APQD)$**



### Solution:

To prove that  $ar(ABCD) = ar(APQD)$

Proof: AS  $AB \parallel DC$  and  $AB \parallel DQ$

In triangle CLQ and triangle BLP,

$$\angle QCL = \angle LBP \quad [\text{Alternate angles}]$$

$$CL = LP \quad [L \text{ is the mid-point of } BC]$$

$$\angle CLQ = \angle BLP \quad [\text{Vertical opposite angles}]$$

$$\triangle CLQ \cong \triangle BLP \quad [\text{By ASA congruence rule}]$$

$$\text{So, } ar(\triangle CLQ) = ar(\triangle BLP) \quad \dots(I) \quad [\text{Congruent triangles are equal in area}]$$

Now, adding  $ar(PLCD)$  both side in above equation, get:

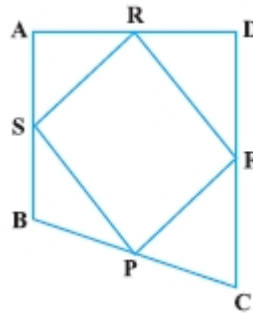
$$ar(\triangle CLQ) + ar(\triangle PLCD) = ar(\triangle BLP) + ar(\triangle PLCD)$$

$$ar(\triangle APQD) = ar(\triangle ABCD)$$

Hence, proved.

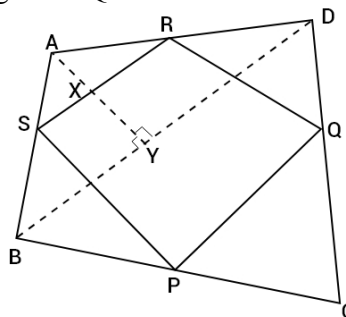
**9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Fig.).**

**[Hint: Join BD and draw perpendicular from A on BD.]**



**Solution:**

According to the question, a quadrilateral ABCD in which the mid-point of the sides of it are joined in order of form parallelogram PQRS.



To Prove that  $ar(PQRS) = \frac{1}{2} ar(ABCD)$

Construction: Join BD and draw perpendicular from A and BD which interest SR and BD at X and Y respectively.

Proof: In triangle ABD, S and R are the mid-points of sides AB and AD respectively. So,  
 $SR \parallel BD$

And:  $ASX \parallel BY$

See the figure, x is the mid-point of AY. So,  
 $AX = XY$

And  $SR = \frac{1}{2} BD \dots (II)$  [mid-point theorem]

Now,  $ar(\triangle ABD) = \frac{1}{2} \times BD \times AY$



$$ar(\Delta ASR) = \frac{1}{2} \times SR \times AX$$

$$ar(\Delta ASR) = \frac{1}{2} \times \left(\frac{1}{2} BD\right) \times \left(\frac{1}{2} AY\right) \quad [\text{Using equation (I) and (II)}]$$

$$ar(\Delta ASR) = \frac{1}{4} \times \left(\frac{1}{2} BD \times AY\right)$$

$$ar(\Delta ASR) = \frac{1}{4} \times (\Delta ABD) \quad \dots \text{(III)}$$

$$\text{Again, } ar(\Delta CPQ) = \frac{1}{4} ar(\Delta CBD) \quad \dots \text{(IV)}$$

$$ar(\Delta BPS) = \frac{1}{4} ar(\Delta BAC) \quad \dots \text{(V)}$$

$$ar(\Delta DRQ) = \frac{1}{4} ar(\Delta DAC) \quad \dots \text{(VI)}$$

Now, adding equation (III), (IV), (V) and (VI), get:

$$\begin{aligned} & ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ) \\ &= \frac{1}{4} ar(\Delta ABD) + \frac{1}{4} ar(\Delta CBD) + \frac{1}{4} ar(\Delta ABC) + \frac{1}{4} ar(\Delta DAC) \\ &= \frac{1}{4} [ar(\Delta ABD) + ar(\Delta CBD) + ar(\Delta ABC) + ar(\Delta DAC)] \\ &= \frac{1}{4} [ar(ABCD) + ar(ABCD)] \\ &= \frac{1}{4} \times 2ar(ABCD) \\ &= \frac{1}{2} ar(ABCD) \end{aligned}$$

$$\text{So, } ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ) = \frac{1}{2} ar(ABCD)$$

$$ar(ABCD) - ar(PQRS) = \frac{1}{2} ar(ABCD)$$

$$\text{Now, } ar(PQRS) = ar(ABCD) - \frac{1}{2} ar(ABCD)$$

$$ar(PQRS) = \frac{1}{2} ar(ABCD)$$

Hence, proved.

## Exercise No. 9.4

### Long Answer Questions:

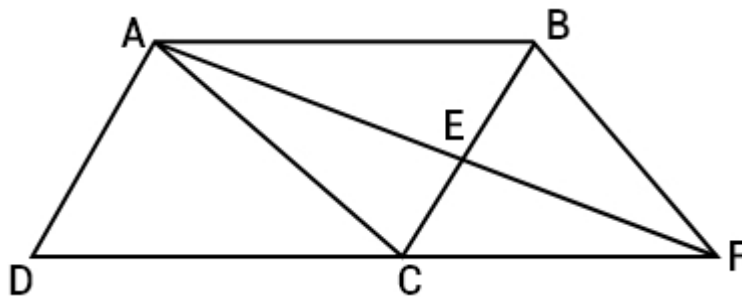
1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that  $\text{ar}(\triangle ADF) = \text{ar}(\triangle ABFC)$

#### Solution:

Given in the question, A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F.

Prove that  $\text{ar}(\triangle ADF) = \text{ar}(\triangle ABFC)$

Proof: ABCD is a parallelogram and AC divides it into two triangles of equal area.



$$\text{ar}(\triangle ADC) = \text{ar}(\triangle ABC)$$

So,  $DC \parallel AB$  and  $CF \parallel AB$

As we know that triangles on the same base and between the same parallels are equal in area.  
So,

$$\text{ar}(\triangle ACF) = \text{ar}(\triangle BCF) \quad \dots(\text{II})$$

Adding equation (I) and (II), get:

$$\text{ar}(\triangle ADC) + \text{ar}(\triangle ACF) = \text{ar}(\triangle ABC) + \text{ar}(\triangle BCF)$$

$$\text{ar}(\triangle ADF) = \text{ar}(\triangle ABFC)$$

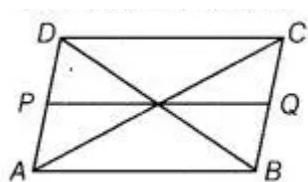
Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. show that PQ divides the parallelogram into two parts of equal area.

#### Solution:

Given: ABCD is a parallelogram and diagonals intersect at O, and draw a line PQ which intersects AD and BC.

To prove that PQ divides the parallelogram ABCD into two parts of equal area that  $ar(ABQP) = ar(CDPQ)$ .



Proof: AC is a diagonal of the parallelogram ABCD.

$$ar\left(\Delta \frac{1}{2} ACD\right) = \frac{1}{2} ar(ABCD) \quad \dots(I)$$

In triangle AOP and triangle COQ,

AO = CO [Diagonals of a parallelogram bisect each other]

$\angle AOP = \angle COQ$  [Vertical opposite angles]

$\angle OAP = \angle OCQ$  [Alternate angles, AB||CD]

$\Delta AOP = \Delta COQ$  [By ASA congruent rule]

Since,  $ar(\Delta AOP) = ar(\Delta COQ)$  [Congruent area axiom]  $\dots(II)$

Now, adding  $ar(AOQD)$  to both sides of (II), get:

$$ar(\Delta ACD) = \frac{1}{2} ar(ABCD) \quad \text{[From equation (I)]}$$

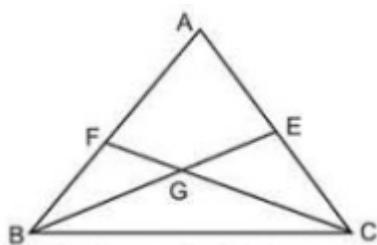
$$\text{Hence, } ar(APQD) = \frac{1}{2} ar(ABCD).$$

**3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of  $\Delta GBC$  = area of the quadrilateral AFGE**

**Solution:**

Given: The medians BE and CF of a triangle ABC intersect at G

To prove that  $ar(\Delta GBC) = ar(AFGE)$ .



Proof: As median CF divides a triangle into two triangle of equal area. So,

$$ar(\Delta BCF) = ar(\Delta ACF)$$

$$ar(\Delta GBF) + ar(\Delta GBC) = ar(AFGE) + ar(\Delta GCE) \quad \dots(I)$$

Now, median BE divides a triangle into two triangle of equal area. So,

$$ar(\Delta GBF) + ar(AFGE) = ar(\Delta GCE) + ar(\Delta GBC) \quad \dots(II)$$

Now, subtracting (II) from (I), get:

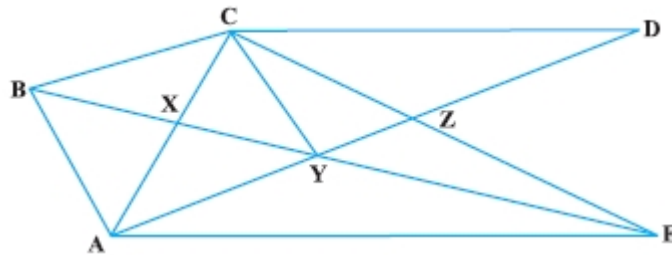
$$ar(\triangle GBC) - ar(\triangle FGE) = ar(\triangle AFG) - ar(\triangle GBC)$$

$$ar(\triangle GBC) + ar(\triangle GBC) = ar(\triangle AFG) + ar(\triangle AFG)$$

$$2ar(\triangle GBC) = 2ar(\triangle AFG)$$

Hence,  $ar(\triangle GBC) = ar(\triangle AFG)$ .

**4. In Fig.,  $CD \parallel AE$  and  $CY \parallel BA$ . Prove that  $ar(\triangle CBX) = ar(\triangle AXY)$**



**Solution:**

Given:  $CD \parallel AE$  and  $CY \parallel BA$

To prove that  $ar(\triangle CBX) = ar(\triangle AXY)$ .

Proof: As we know that triangle on the same base and between the same parallels are equal in area. So,

$$ar(\triangle ABC) = ar(\triangle ABE)$$

$$ar(\triangle CBX) + ar(\triangle ABX) = ar(\triangle ABX) + ar(\triangle AXY)$$

Hence,  $ar(\triangle CBX) = ar(\triangle AXY)$ .

**5. ABCD is a trapezium in which  $AB \parallel DC$ ,  $DC = 30$  cm and  $AB = 50$  cm. If X and Y are, respectively the mid-points of AD and BC, prove that**

$$ar(\triangle DCY) = \frac{7}{9} ar(\triangle XYBA)$$

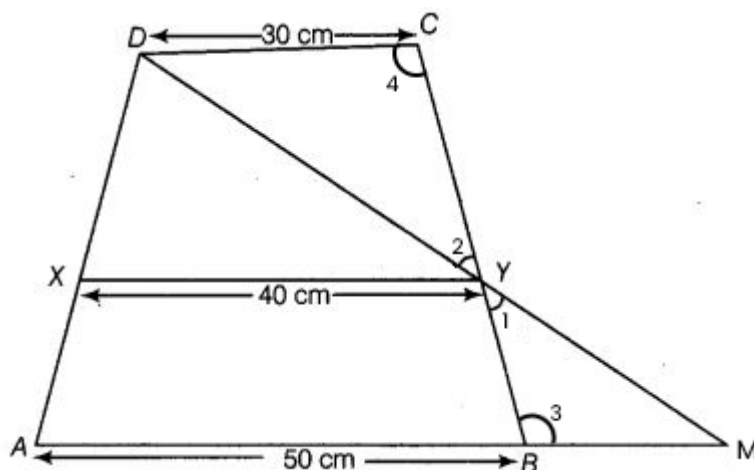
**Solution:**

To prove that  $ar(\triangle DCY) = \frac{7}{9} ar(\triangle XYBA)$

Proof: In triangle MBY and triangle DCY,

$$\angle 1 = \angle 2 \quad [\text{Vertically opposite angles}]$$

$$\angle 3 = \angle 4 \quad [AB \parallel DC \text{ and alternate angles are equal}]$$



$BY = CY$  [Y is the mid-point of BC]  
 $\triangle MBY \cong \triangle DCY$  [By ASA congruent angle]  
 So,  $MB = DC = 30 \text{ cm}$  [CPCT]  
 Now,  $AM = AB + BM$   
 $= 50 \text{ cm} + 30 \text{ cm}$   
 $= 80 \text{ cm}$

In triangle ADM,

$$XY = \frac{1}{2} AM = \frac{1}{2} \times 80 \text{ cm} = 40 \text{ cm}$$

Now,  $AB \parallel XY \parallel DC$  and X and Y are the mid-points of AD and BC, So, height of trapezium DCXY and XYBA are equal and assume the equal height be h cm.

$$\frac{\text{ar}(DCYX)}{\text{ar}(XYBA)} = \frac{\frac{1}{2} \times (30 + 40) \times h}{\frac{1}{2} \times (30 + 50) \times h} = \frac{70}{90} = \frac{7}{9}$$

Hence,  $\text{ar}(DCYX) = \frac{7}{9} \text{ar}(XYBA)$ .

**6. In  $\triangle ABC$ , if L and M are the points on AB and AC, respectively such that  $LM \parallel BC$ . Prove that  $\text{ar}(\text{LOB}) = \text{ar}(\text{MOC})$**

**Solution:**

Given: in triangle ABC and L and M are the points on AB and AC, respectively such that  $LM \parallel BC$ .

Prove that  $\text{ar}(\text{LOB}) = \text{ar}(\text{MOC})$

Proof: As we know that triangle on the same base and between the same parallels are equal in area.

$$\text{ar}(\triangle LBM) = \text{ar}(\triangle LCM)$$

[Triangle LBM and triangle LCM are on the same base LM and between the same parallels LM and BC]

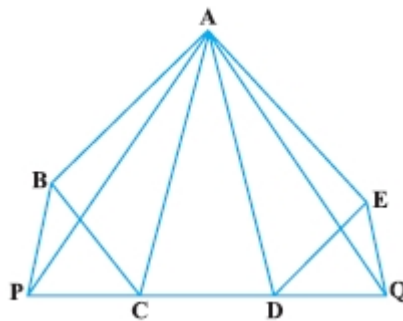
$$ar(\triangle LBM) = ar(\triangle LCM)$$

$$ar(\triangle LOM) + ar(\triangle LOB) = ar(\triangle LOM) + ar(\triangle MOC)$$

Hence,  $ar(\triangle LOB) = ar(\triangle MOC)$ .

**7. In Fig., ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that**

$$ar(ABCDE) = ar(APQ)$$



**Solution:**

Given: ABCDE is any pentagon and BP  $\parallel$  AC meets DC produced at P and EQ  $\parallel$  AD meets CD produced at Q.

Prove that  $ar(ABCDE) = ar(APQ)$

Proof: As we know that triangle on the same base and between the same parallels are equal in area.

$$ar(\triangle ABC) = ar(\triangle APC) \quad \dots(I)$$

$$ar(\triangle ADE) = ar(\triangle ADQ) \quad \dots(II)$$

Now, adding equation (I) and (II), get:

$$ar(\triangle ABC) + ar(\triangle ADE) = ar(\triangle APC) + ar(\triangle ADQ)$$

Now, adding  $ar(\triangle ACD)$  to both side, get:

$$ar(\triangle ABC) + ar(\triangle ADE) + ar(\triangle ACD) = ar(\triangle APC) + ar(\triangle ADQ) + ar(\triangle ACD)$$

Hence,  $ar(ABCDE) = ar(APQ)$ .

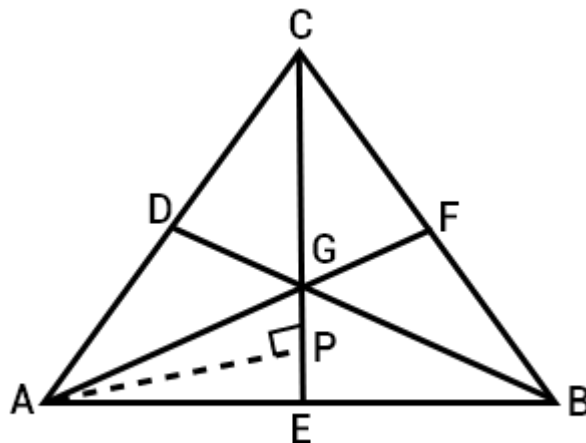
**8. If the medians of a  $\triangle ABC$  intersect at G, show that**

$$ar(AGB) = ar(AGC) = ar(BGC) = \frac{1}{3} ar(ABC)$$

**Solution:**

Given: The median of a triangle ABC intersect at G.

To prove that  $\text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$



Construction: Draw  $BP \perp EG$

Proof:  $AG = \frac{2}{3} AE$  [Centroid divides the median in the ratio 2:1]

Now,  $\text{ar}(\triangle AGB) = \frac{1}{2} \times AG \times BP$

[Median divides a triangle into two triangles equal in area]

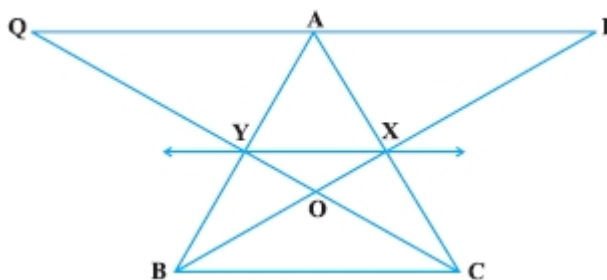
$= \frac{1}{3} \text{ar}(\triangle ABC)$

Again,  $\text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$

So,  $\text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$

Hence, proved.

**9. In Fig., X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that  $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$ .**



**Solution:**

Given: In triangle ABC, X and Y are the mid-points of AB and AC.

To prove that  $ar(\triangle ABP) = ar(\triangle ACQ)$ .

Proof: Since,  $XY \parallel BC$  [BY mid-point theorem]

As we know that triangle on the same base and between the same parallels lines are equal in area. So,

$$ar(\triangle BYC) = ar(\triangle BXC) \quad \dots(I)$$

Now, subtracting  $ar(\triangle BOC)$  from both sides in the above, get:

$$ar(\triangle BYC) - ar(\triangle BOC) = ar(\triangle BXC) - ar(\triangle BOC)$$

$$ar(\triangle BOY) = ar(\triangle COX) \quad \dots(II)$$

Now, adding  $ar(\triangle XOY)$  to both side in equation (II), get:

$$ar(\triangle BOY) + ar(\triangle XOY) = ar(\triangle COX) + ar(\triangle XOY) \quad \dots(III)$$

Again, quadrilaterals XYAP and YXAQ are on the same base XY and between the same parallels XY and PQ. So,

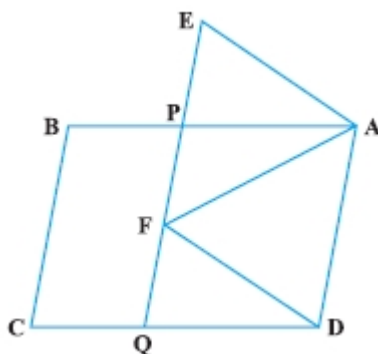
$$ar(\triangle XYAP) = ar(\triangle XYQA) \quad \dots(IV)$$

Now, adding equation (III) and (IV), get:

$$ar(\triangle BXY) + ar(\triangle XYAP) = ar(\triangle CXY) + ar(\triangle XYAQ)$$

Hence,  $ar(\triangle ABP) = ar(\triangle ACQ)$ .

**10. In Fig., ABCD and AEFD are two parallelograms. Prove that  $ar(\triangle PEA) = ar(\triangle QFD)$  [Hint: Join PD].**

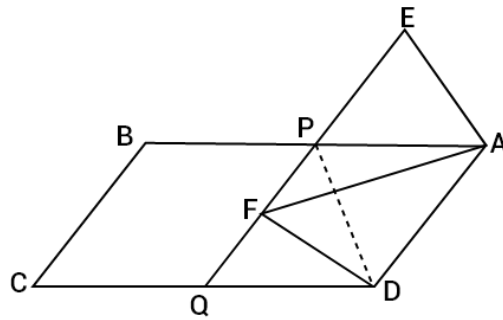
**Solution:**

Given: ABCD and AEFD are two parallelogram.

To prove that  $ar(\triangle PEA) = ar(\triangle QFD)$

Construction: join PD.





Proof: In triangle PEA and triangle QFD,

$\angle APE = \angle DQF$  [Corresponding angles are equal as  $AB \parallel CD$ ]

$\angle AEP = \angle DEQ$  [Corresponding angles are equal as  $AE \parallel DF$ ]

$AE = DF$  [Opposite sides of a parallelogram are equal]

So,  $\triangle PEA \cong \triangle QFD$  [By AAS congruent rule]

Hence,  $ar(\triangle PEA) = ar(\triangle QFD)$ .